Study material on Radioactivity
SEM – IV   CC – IX

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What is Radioactivity?

- Radioactivity is the phenomenon exhibited by the nuclei of an atom as a result of nuclear instability. It is a process by which the nucleus of an unstable atom loses energy by emitting radiation.
- The nucleus of an atom is held together by the constant balance of two forces; strong nuclear forces of attraction and electrostatic forces of repulsion. These are the two strongest forces in nature.
- Since holding that much-concentrated mass in one tiny nucleus is a very challenging task, as we increase the size of the nucleus, the chances of instability are higher. Heavy nuclei of atoms such as Uranium, Plutonium are very prone to radioactivity.

Radioactive Decay:

Radioactivity is basically the process of decomposition of nuclei. The unstable atomic nuclei spontaneously decompose to form nuclei with a higher stability. This process which occurs for the attainment of stability is called radioactivity. The process leads to the emission of energy and particles which are called radiation. There can be two kinds of radioactivity- natural or induced radioactivity. Henry Becquerel discovered this phenomenon. It is a process by which the nucleus of an unstable atom loses energy by emitting radiation.

Natural Radioactivity: When unstable nuclei decompose in nature, it is termed as natural radioactivity.

Induced Radioactivity: As the name suggests, when unstable nuclei are prepared in the laboratory, the decomposition is termed as the induced radioactivity.

Radioactive Decay Law:

According to the law of radioactive decay, when a radioactive material undergoes either α or β or γ decay, the number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the given sample material. The mathematical representation of the law of radioactive decay is:

$-\frac{\Delta N}{\Delta t} \propto N$ (The minus sign indicates the number of original nuclei decreases over time.)

Where, N = the total number of nuclei in the sample

ΔN = number of nuclei that undergoes decay

Δt = time interval

$$\frac{\Delta N}{\Delta t} = \lambda N \qquad \text{(eq.1)}$$

Where, λ is radioactive decay constant also known as disintegration constant

The change in the sample with respect to the number of nuclei is given as:

$$\frac{dN}{dt} = -\lambda N$$

$$\text{or, } \frac{dN}{N} = -\lambda dt$$

Integrating both sides of the equation, and defining N_0 to be the number of nuclei at $t = 0$,

$$\int_{N_0}^{N} \frac{\partial N}{N} = -\lambda \int_0^t \partial t \quad \text{(eq.2)}$$

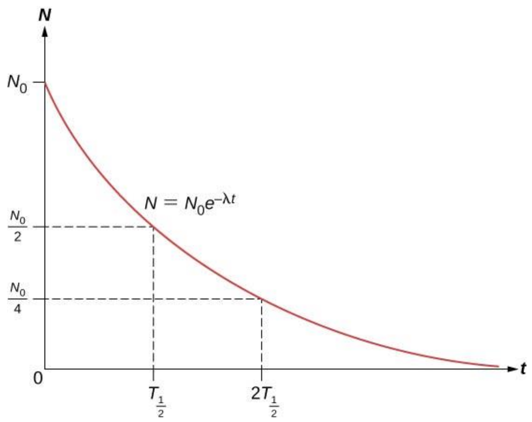
$$\text{or, } \ln N - \ln N_0 = -\lambda(t - t_0)$$

$$\text{or, } \ln\left(\frac{N}{N_0}\right) = -\lambda t \quad \text{(eq 3)}$$

$$\text{or, } N(t) = N_0\, e^{-\lambda t} \quad \text{(eq.4)}$$

This is called **radioactive decay Law.**

The total number of nuclei drops very rapidly at first, and then more slowly (Figure 1).



Half Life: The half-life ($T_{1/2}$) of a radioactive substance is defined as the time for half of the original nuclei to decay (or the time at which half of the original nuclei remain). The number of radioactive nuclei remaining after an integer (n) number of half-lives is therefore

$$N = \frac{N_0}{2^n} \quad \text{(eq 5)}$$

If the decay constant (λ) is large, the half-life is small, and vice versa. To determine the relationship between these quantities, note that when $t = T_{1/2}$, then $N = \frac{N_0}{2}$.

Thus, Equation 5 can be rewritten as

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \quad \text{(eq 6)}$$

Dividing both sides by N_0 and taking the natural logarithm yields

$$\ln\frac{1}{2} = \ln e^{-\lambda t_{1/2}} \quad \text{(eq 7)}$$

which reduces to

$$\lambda = \frac{0.693}{t_{1/2}} \quad \text{(eq 8)}$$

Thus, if we know the half-life $T_{1/2}$ of a radioactive substance, we can find its decay constant.

Average Life or Mean Life: The lifetime $\overline{T}$ of a radioactive substance is defined as the average amount of time that a nucleus exists before decaying. The lifetime of a substance is just the reciprocal of the decay constant, written as

$$\overline{T} = 1/\lambda. \quad \text{(eq 9)}$$

The activity A is defined as the magnitude of the decay rate,

$$A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \quad \text{(eq 10)}$$

The infinitesimal change dN in the time interval dt is negative because the number of parent (undecayed) particles is decreasing, so the activity (A) is positive. Defining the initial activity as $A_0 = \lambda N_0$, we have

$$A = A_0 e^{-\lambda t} \quad \text{(eq 15)}$$

Thus, the activity *A* of a radioactive substance decreases exponentially with time (Figure 2).

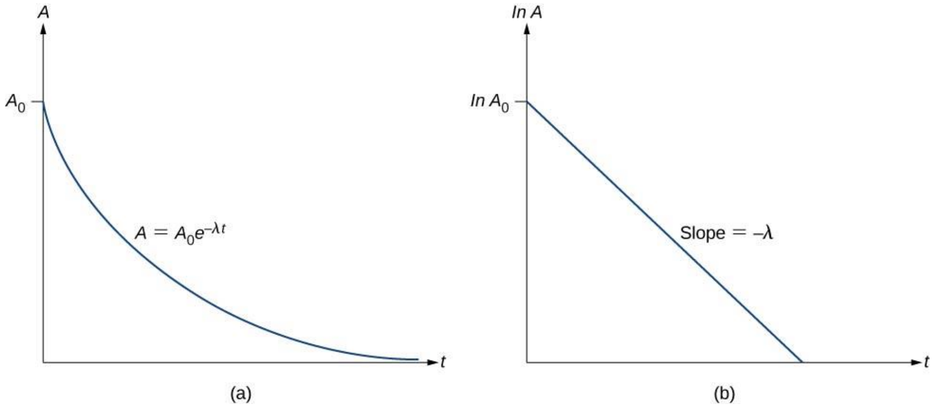


Figure 2: (a) A plot of the activity as a function of time (b) If we measure the activity at different times, we can plot ln A versus t, and obtain a straight line.

Expressing λ in terms of the half-life of the substance, we get

$$A = A_0 e^{(0.693/T_{1/2})\,T_{1/2}} = A_0\, e^{0.693} = A_0/2. \quad \text{(eq 16)}$$

Therefore, the activity is halved after one half-life. We can determine the decay constant λ by measuring the activity as a function of time. Taking the natural logarithm of the left and right sides of Equation 16 , we get

$$\ln A = -\lambda t + \ln A_0 \quad \text{(eq 17)}$$

This equation follows the linear form y = mx+b. If we plot ln A versus t, we expect a straight line with slope (−λ) and y-intercept $\ln A_0$ (Figure 2b). Activity A is expressed in units of becquerels (Bq), where one 1Bq = 1decay per second. This quantity can also be expressed in decays per minute or decays per year. One of the most common units for activity is the curie (Ci), defined to be the activity of 1 g of ^{226}Ra. The relationship between the Bq and Ci is

$$1\text{Ci} = 3.70\times10^{10}\ \text{Bq}$$

Types of Radioactive Decay:

The spontaneous decay or breakdown of an atomic nucleus is known as Radioactive Decay. This decay in a nucleus causes the release of energy and matter from the nucleus. The most common forms of Radioactive decay are:

1) Alpha Decay (Helium nucleus is emitted)
2) Beta Decay (Electrons are emitted)
3) Gamma Decay (High energy photons are emitted)

This is also termed as Nuclear Decay or Radioactivity. The element or isotope which emits radiation and undergoes the process of radioactivity is called Radioactive Element.

1) Alpha decay:

Alpha rays are the positively charged radiation. These rays are more massive than the beta rays and they contain a stream of positively charged particles, called alpha particles, which have the atomic mass of 4 and a positive charge of 2 (a helium nucleus). The emission of an alpha particle from the nucleus decreases the mass number of the nucleus by 4 and the atomic number reduces by 2. When an alpha particle emits its nucleus, the process is called alpha decay.

In general, any alpha decay formula can be written in the following way-

$$^{A}X_{Z} \longrightarrow {}^{A-4}X'_{Z-2} + {}^{4}\alpha_{2}$$

Where,
$^{A}X_{Z}$ is the parent nucleus.

$^{A-4}X'_{Z-2}$ is the daughter nucleus.

$4\alpha_2$ is the emitted alpha particle.

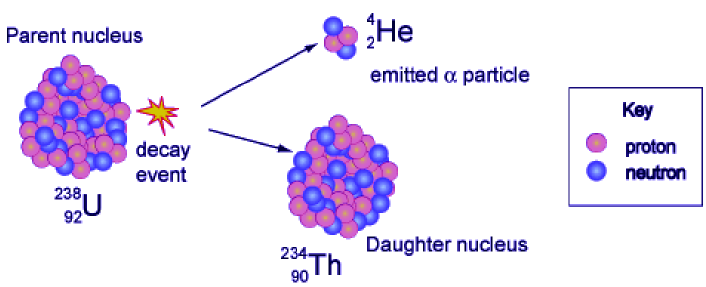
The nucleus of helium is taken as the alpha particle which is very stable. It has a group of two protons and two neutrons. For example, alpha decay of uranium-238 is shown below

$$^{238}U_{92} \rightarrow {}^{234}Th_{90} + {}^{4}He_{2}$$

Where,

- $^{238}U_{92}$ is the unstable Uraniumum-238 parent nucleus.
- $^{234}Th_{90}$ is the stable Thorium-234 daughter nucleus.
- $^{4}He_{2}$ is the emitted alpha particle.

Alpha Decay of a Uranium-238 nucleus



The alpha radiations are made up of two neutrons and two protons which are attached to the nucleus of a helium atom. These particles have an intense ionization power which means that the moment they come into contact with atoms of a living tissue they have the ability of causing mutations and at times such reactions may even lead to cancer. Despite of their high ionization power these rays are not very harmful. In fact, out of the three rays, these are the least dangerous as long as they are not inhaled. Alpha rays cannot penetrate through the skin and can even be stopped by a few centimetres of air.

Uses of Alpha Radiation:

There are many ways in which science successfully uses alpha radiation in a beneficial way.

- Cancer Treatment - Alpha radiation is used to treat various forms of cancer.

- Static Eliminator - Alpha radiation from polonium-210 is used to eliminate static electricity in industrial applications.
- Smoke Detector
- Spacecraft Power
- Pacemaker Battery - Plutonium-238 is used as the fuel source for such batteries; with a half-life of 88 years, this source of power provides a long lifespan for pacemakers.
- Remote Sensing Stations
- Heating Devices
- Coast Guard Buoys
- Oil Well Equipment
- Seismic and Oceanographic Devices

2) Beta Decay:

Beta Decay is a type of radioactive decay in which a proton is transformed into a neutron or vice versa inside the nucleus of the radioactive sample. Processes like this and alpha decay allow the nucleus of the radioactive sample to get as close as possible to the optimum neutron/ proton ratio. While doing so, the nucleus emits a beta particle which can either be an electron or positron. Remember that there either a proton can turn to a neutron or neutron to a proton. Electron and the positron are generated to obey the law of conservation of charge. The beta decay occurs via the weak interaction. There are two types of beta decay, namely, beta minus (β-) and beta plus (β+).

Beta-Minus decay:

- In beta minus, a neutron is transformed to yield a proton causing an increase in the atomic number of the atom. The neutron is neutral but the proton is positive.
- To maintain conservation of charge, the nucleus in the process also produces an electron and an antineutrino.
- Antineutrino is the antimatter counterpart of neutrino. Both of these are neutral particles with little mass. They interact with matter very weakly and can even pass through the entire earth without being disturbed.
- In a beta minus decay, the change in atomic configuration is;
  $^{A}X_{Z} \rightarrow {}^{A}Y_{Z+1} + e^{-} + \overline{\nu}$
  $N = p + e^{-} + v^{-}$

Beta-Plus Decay:

- In beta plus decay, the proton disintegrates to yield a neutron causing a decrease in the atomic number of the radioactive sample. The nucleus experiences a loss of proton but gains a neutron.
- Again, conservation of charge is important. The beta plus decay in order to obey the conservation law also yields a positron and a neutrino.
- A positron is the antimatter equivalent of an electron; the same in all aspects except that a positron has a positive charge.

- A Neutrino’s behaviour is the same as the antineutrino’s. Expressed in the equation it is,

  $^{A}X_{Z} \rightarrow {}^{A}Y_{Z-1} + e_{+} + \nu$

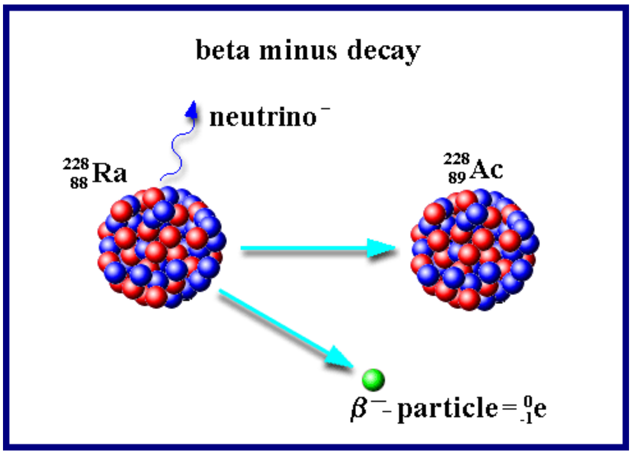
  $P = n + e^{+} + v$

  Beta plus decay can happen only if the daughter nucleus is more stable than the mother nucleus. This difference goes into the conversion of a proton into a neutron, a positron and a neutrino. There is no increase in mass number because a proton and a neutron have the same mass.

  Beta radiations consist of a stream of electrons termed as beta particles. On the emission of a beta particle, a neutron in the nucleus gets converted to a proton and hence the mass number remains unchanged but the atomic number increases by one unit. Many beta emitters occur naturally in the radioisotopes found in the natural radioactive decay chains of uranium, thorium and actinium. Examples include lead-210, bismuth-214 and thallium-206. Consider the example given below in which the electron is the beta particle:

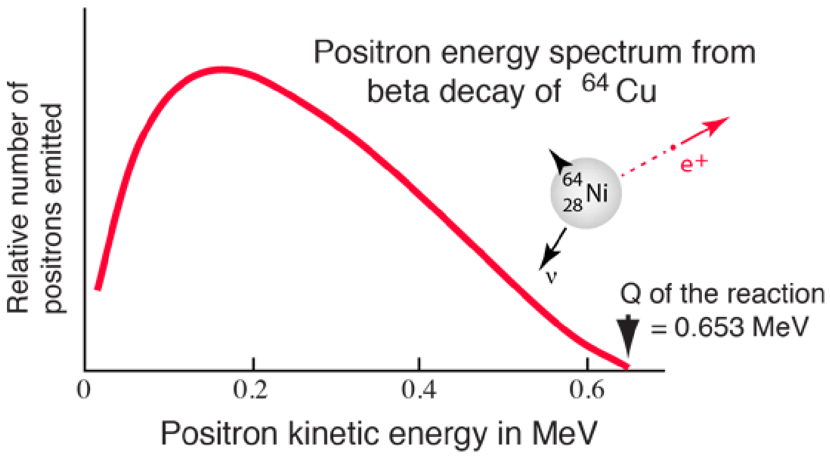
$$^{234}\mathbf{Th}_{90} \rightarrow {}^{234}\mathbf{Pa}_{91} + {}^{0}\mathbf{e}_{-1}$$

Beta rays are made up of high energy electrons. They are less ionizing than the alpha rays but are more harmful as they can penetrate through the skin. They can be stopped with an aluminium sheet.



Beta Energy Spectrum

In the process of beta decay, either an electron or a positron is emitted. Because either a neutrino or an antineutrino is emitted as well, there is a spectrum of energies for the electron or positron, depending upon what fraction of the reaction energy Q is carried by the massive particle. The shape of this energy curve can be predicted from the Fermi theory of beta decay.

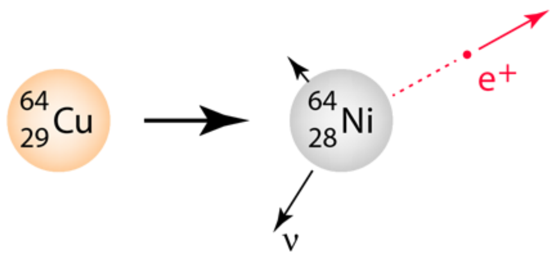


Electron and Antineutrino:

Studies of beta decay revealed a continuous energy spectrum up to a maximum, unlike the predictable energy of alpha particles. Another anomaly was the fact that the nuclear recoil was not in the the direction opposite the momentum of the electron. The emission of another particle was a probable explanation of this behavior, but searches found no evidence of either mass or charge. The interesting history has Wolfgang Pauli in 1930 proposing an as yet unobserved particle to explain the continuous distribution of energy of the emitted electrons. Then Enrico Fermi called this particle a neutrino and developed a theory of beta decay in which the neutrino carried away the missing energy and momentum. With no charge and almost no mass, it was hard to detect, and not until 1956 was experimental detection of the neutrino achieved. For symmetry reasons, the particle emitted along with the electron from nuclei is called an antineutrino. The emission of a positron is accompanied by a neutrino.

Positron and Neutrino:

The emission of a positron or an electron is referred to as beta decay. The positron is accompanied by a neutrino, an almost massless and chargeless particle. Positrons are emitted with the same kind of energy spectrum as electrons in negative beta decay because of the emission of the neutrino.



Some applications of beta particles include:

- Treatment eye and bine cancer
- Used in phosphorescent lighting typically for emergency lighting as it requires no power
- Used as thickness detectors for the quality control of thin materials such as paper

3) Gamma Decay:

Gamma decay is the emission of electromagnetic radiation of an extremely high frequency i.e. very high energy, giving out excess energy in order to stabilize the unstable nucleus. The Nucleus too has its own energy levels. Gamma decay is the nucleus's way of dropping from a higher energy level to a lower energy level through the emission of high energy photons. The energy level transition energies in the atom are in the order of MeV. Therefore, the gamma-ray emitted is also of very high energy of the order of MeV, just like x-rays. The gamma rays emitted can be differentiated from x-rays only by the fact that gamma rays come from the nucleus. Due to their high energy, they are extremely penetrating and thereby dangerous to biological life forms.

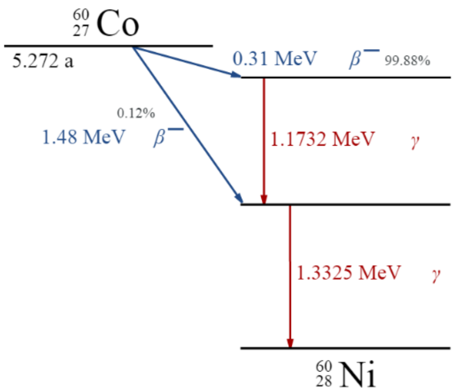
Here we will speak a little further about the distinction between an x-ray and a gamma-ray. X-rays are emitted by electrons (either in the orbits or in outside applications like particle accelerators, synchrotrons radiation etc) whereas gamma rays are emitted by the nucleus, particle decay or annihilation reactions.

Unlike, alpha decay and beta decay, the parent nucleus does not undergo any physical change in the process, daughter and parent nuclei are the same. Most of the time, gamma decay occurs after the radioactive nuclei have undergone an alpha or a beta decay. The alpha and beta decays leave the daughter nuclei in an excited state. From the excited state, the daughter nuclei can get back to the ground state by emitting one or more high energy gamma rays. The γ decay equation is simply

$$^{A}_{Z}X^{*}_{N} \rightarrow X_N + Y_1 + Y_2 \ldots (\gamma \text{ decay})$$

where the asterisk indicates the nucleus is in an excited state. There may be one or more γ s emitted, depending on how the nuclide de-excites. In radioactive decay, γ emission is common and is preceded by γ or β decay. For example, when ^{60}Co $\beta-$ decays, it most often leaves the daughter nucleus in an excited state, written ^{60}Ni*. Then the nickel nucleus quickly γ decays by the emission of two penetrating γs: $^{60}\text{Ni}^* \rightarrow {}^{60}\text{Ni} + \gamma_1 + \gamma_2$. These are called cobalt γ rays, although they come from nickel—they are used for cancer therapy, for example. It is again constructive to verify the conservation laws for gamma decay.

Let's look at an example:



The image above shows the path taken by Cobalt-60 to move from an excited state to a non-excited state. The beta decay can leave it at either one of the two energy levels. The percentages mentioned next to the beta symbol is the probability of nuclei choosing either of the two paths. Beta-decay followed by a gamma decay.

$$^{60}_{27}Co \rightarrow ^{60}_{28}Ni + e^- + \nu + \gamma + 1.17\ MeV$$

$$^{60}_{28}Ni^* \rightarrow ^{60}_{28}Ni + \gamma + 1.33\ MeV$$

Gamma radiations are extremely high energy photons with a very short wavelength (0.0005 to 0.1 nm). Alpha and beta emission are often accompanied by gamma emission, as an excited nucleus drops to a lower and more stable energy state. Gamma rays are electromagnetic waves of high frequency with no mass and no charge. They have the least ionizing power but are most dangerous. Gamma rays have the highest penetrating power and can only be stopped by a few centimetres of lead or few meters of concrete. In certain cases they may even pass through them.

The nucleus has orbiting electrons which indeed have some energy, and when an electron jumps from a level of high energy to a level of low energy, there is an emission of a photon. The same thing happens in the nucleus: whenever it rearranges into a lower energy level, a high-energy photon is shooted out which is known as a gamma ray.

Sources of Gamma Rays:

Sources of gamma rays other than radioactive decay include terrestrial thunderstorms and lightning, from celestial bodies such as pulsars, quasars, distant galaxies, gamma-ray bursts in space and collapse of a star into a black hole known as a hypernova aka super-luminous supernova. Hypernova events result in bursts of long-duration gamma-ray emissions. These emissions produce a total energy output of about 1044 Joules (as much energy as our Sun will produce in an entire lifetime) in a span of 20-40 seconds.

Gamma rays cause damage on a cellular level and due to their penetrating nature, they can diffuse this damage through the entire body. However, gamma rays are less ionizing that alpha or beta hence the severity is lesser, but penetration is more.

Applications of Gamma Rays:

Some of the most energetic phenomena in the universe occurs through gamma rays. We cannot witness these events without a gamma-ray detector. To address this, scientists have created a satellite called Fermi Gamma-ray Space Telescope that provides an unparalleled view of the universe. Gamma-ray sensors are also used in the food packaging and chemical industry to measure density, thickness and composition. Gamma rays are used to treat certain types of cancer where the high energy gamma beams are irradiated on the cancerous cells to kill them.

Energy and momentum conservation

- Gamma-ray decay is a binary decay.
- Gamma-rays are massless but they carry (relatively small) momentum.

$$\mathbf{E_\gamma = p_\gamma c \Rightarrow p_\gamma = E_\gamma / c} \qquad (1)$$

- For the parent at rest conservation of momentum implies that the daughter nucleus recoils with the momentum of the same magnitude but in the opposite direction

$$\vec{p_d} + \vec{p_\gamma} = \mathbf{0} \Rightarrow \vec{p_d} = -\vec{p_\gamma} \qquad (2)$$

- The recoil energy is small thus it can be calculated non-relativistically.
- The energy conservation for the initial state of mass M_i and the final state of mass M_f in the γ-decay is

$$\mathbf{M_i c^2 = M_f c^2 + E_\gamma + T_d} \qquad (3)$$

- The energy conservation can be rewritten as

$$\mathbf{M_i c^2 = M_f c^2 = E_i - E_f = \Delta E = E_\gamma + T_d} \qquad (4)$$

with ΔE being the energy difference between the initial and the final state.

- Note that the final state does not have to be the ground state. The decay can very well go between excited states.
- If the initial state is high-energy cascades of several γ rays are emitted before the ground state is reached.
- Note that the energy of the γ ray is smaller than the energy difference between excited states since part of the energy is in the recoil of the daughter. This is significant and crucial for understanding the M¨ossbauer effect.
- The recoil energy of the daughter is

$$\mathbf{T_d} = \frac{p_d^2}{2M_d} = \frac{p_\gamma^2}{2M_d} = \frac{E_\gamma^2}{2M_d c^2} \qquad (5)$$

- The energy of the γ-ray can be calculated from

$$\mathbf{\Delta E = E_\gamma + T_d = E_\gamma} + \frac{E_\gamma^2}{2M_d c^2} \qquad (6)$$

- The γ-ray energy is

$$\mathbf{E_\gamma} = \left(\sqrt{\frac{2\nabla E}{M_d c^2} + 1} - 1\right) \mathbf{M_d c^2} \approx \Delta E \left(1 - \frac{1}{2}\frac{\nabla E}{M_d c^2}\right) \qquad (7)$$

- The above reasoning implicates that the γ ray energy is lower than the energy difference between excited states by

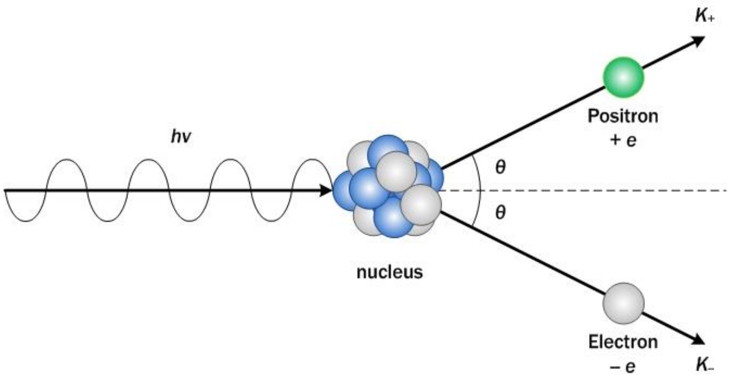
$$\mathbf{E\gamma = \Delta E - \delta \Rightarrow \delta = \Delta E}\, \frac{1}{2}\frac{\nabla E}{M_d c^2}$$

Pair Production by γ photon:

Pair production, in physics, formation or materialization of two electrons, one negative and the other positive (positron), from a pulse of electromagnetic energy traveling through matter occurs usually in the vicinity of an atomic nucleus. Pair production is a direct conversion of radiant energy to matter. It is one of the principal ways in which high-energy gamma rays are absorbed in matter. For pair production to occur, the electromagnetic energy, in a discrete quantity called a photon, must be at least equivalent to the mass of two electrons. The mass m of a single electron is equivalent to 0.51 million electron volts (MeV) of energy E as calculated from the equation formulated by Albert Einstein, $E = mc^2$, in which c is a constant equal to the velocity of light. To produce two electrons, therefore, the photon energy must be at least 1.02 MeV. Photon energy in excess of this amount, when pair production occurs, is converted into motion of the electron-positron pair. If pair production occurs in a track detector, such as a cloud chamber, to which a magnetic field is properly applied, the electron and the positron curve away from the point of formation in opposite directions in arcs of equal curvature. In this way pair production was first detected (1933). The positron that is formed quickly disappears by reconversion into photons in the process of annihilation with another electron in matter.

In addition to the photoelectric effect (photon absorption) and Compton scattering (photon scattering), there is a third process by which photons can lose energy in their interaction with matter. In this process, termed *pair production*, a photon can simply vanish and in its place a matter-antimatter pair of particles can appear. This phenomenon is a wonderful illustration of the fact that *mass is not conserved*, since the mass of the electron and positron can be created from the energy of the massless photon. Of course, the photon must have sufficient energy to create the rest masses of the two new particles.

Typically, this process occurs in the vicinity of a nucleus and an electron-positron pair is formed. The effect is sketched below:



The high-energy photon that has energy hv loses its entire energy when it collides with nucleus. Then, it makes pair of electron and positron and gives kinetic energy to each particle.

First, let’s try to imagine a simpler version of this phenomenon, with no nucleus present and the electron and positron both traveling in the same direction as the initial photon.

Energy conservation would lead to:

$$E_{photon} = E_{electron} + E_{positron} \qquad (1)$$

and x-momentum conservation:

$$(pc)_{photon} = (pc)_{electron} + (pc)_{positron} \qquad (2)$$

Using

$$E_{total}^2 = (pc)^2 + (mc^2)^2 \qquad (3)$$

energy conservation can be written as:

$$\sqrt{(pc)_{photon}^2 + (0)^2} = \sqrt{(pc)_{electron}^2 + (0.511)^2} + \sqrt{(pc)_{positron}^2 + (0.511)^2}$$

$$\text{or, } (pc)_{photon} = \sqrt{(pc)_{electron}^2 + (0.511)^2} + \sqrt{(pc)_{positron}^2 + (0.511)^2}$$

Setting this equation equal to the momentum conservation equation leads to:

$$\sqrt{(pc)_{electron}^2 + (0)^2} + \sqrt{(pc)_{electron}^2 + (0)^2} = (pc)_{electron} + (pc)_{positron}$$

Hopefully it's clear that this equation is garbage! The left-side of the equation is larger than the right-side of the equation for any real values of momentum. This means that the "simplified" version of pair production discussed above is impossible. The only way pair production can occur is if a third body (the nucleus) is present to participate in the sharing of energy and momentum.

To be more specific, for pair production to take place, there must be another particle in the vicinity of the photon to ensure momentum conservation. The process in the vicinity of a nucleus can be represented as

$$\Gamma + X \rightarrow e + e^{+} + X^{*},$$

where X and X^* represent the ground and excited states of the nucleus.

Basically, these interactions are ruled by three kind of the law of conservation: total energy, momentum, and electric charge. After the collision, pair of electrons and positron occurs. In this collision, the positron, +e, as a particle, has the same properties which electron has, except its charge sign; the two particle, electron and positron has the opposite charge, and thus its magnetic momentum sign is also opposite. Having an opposite charge sign means that the sum of net charge of pairs is zero, which is actually equal to photon before the collision. Therefore, the conservation of electric charge will be conserved.

Now, what counts is conservation of total energy and momentum. First of all, momentum in this process can be ignored because atomic nucleus is thousands of times more massive than pair of electron and positron, and thus, the photon momentum can be absorbed; thus, it is possible to anticipate that absorbing momentum occur without absorbing much energy. So, it is can be represented by an equation that shows the conservation of total energy only:

$$hv = E_- + E_+ = (m_0c^2 + K_-) + (m_0c^2 + K_+) = K_- + K_+ + 2m_0c^2 \text{ ......eq.1}$$

Here, E_- and E_+ represent total energy of the electron and positron, and K_- and K_+ represent the kinetic energy of the electron and positron. Also, m_0c^2 = 0.511 MeV, and it is the rest mass energy of an electron, which is equal to that of the positron. That is, both the two particles have the same amount of rest mass energy, and from this, we can get the sum, $2m_0c^2$ = 1.02 MeV (1 MeV = 10^6 eV). In the kinetic energy, because of the Coulomb interaction between positively-charged nucleus and both the two particles—the positron and electron, positron would be accelerated and electron would be decelerated; thus, the kinetic energy of positron is actually little larger than electron's.

Intuitively, we can say that since energy is being converted into two particles that have masses, enough energy should be available for this process to take place. That is, the photon must have the energy equivalent of at least the rest masses of two electrons (electron and positron have equal masses).

$$E_{\gamma,threshold} \geq 2m_0c^2$$

$$\Rightarrow E_{\gamma,threshold} \geq 1.022\text{MeV}$$

Here m_e is the mass of an electron or a positron. Hence a photon carrying energy below 1.022 MeV cannot convert into an electron–positron pair.

The actual threshold energy for the process in the vicinity of a heavy nucleus is given by

$$E_{\gamma,threshold} \geq 2m_0c^2 + \frac{2m_0^2c^2}{m_{nuc}}$$

Here m_{nuc} is the mass of the nucleus. We can also write the above equation in the form

$$E_{\gamma,threshold} \geq 2m_0c^2[1 + \frac{m_0}{m_{nuc}}]$$

Since the mass of a nucleus is much greater than the mass of an electron ($m_{nuc} \gg m_o$), we can neglect the second term in the parentheses on the right-hand side and get the threshold condition we found earlier, i.e.,

Pair production can also occur in the vicinity of lighter particles, such as electrons. The process in the vicinity of an electron is generally referred to as *triplet production* and can be written as

$$E_{\gamma,threshold} \geq 2m_0c^2$$

If the photon energy before colliding to nucleus were exactly same with $2m_0c^2$, namely, if the photon energy were 1.02 MeV, the two particles would be created at rest with zero kinetic energy. In this case, we are able to think that the complete conversion of energy into mass occurs in this process. However, if the photon energy were large than 1.02 MeV, then kinetic energy would occur. Also, the process cannot occur when photon energy is below 1.02 MeV. Consequently, From equation 1, we are able to get minimum photon energy that need to produce pair electron and positron, and "$2m_0c^2$ = 1.02 MeV" would be a kind of threshold energy for the pair production that are able to decide whether the pair production process can occur or not, and further, if it occurs, how much kinetic energy each particle has.

The wavelength of $2m_0c^2$ is 0.012Å. If the wavelength of photon is shorter than 0.012Å, it has larger than threshold energy, and total photon energy convert into kinetic energy as well as rest mass energy. This wavelength, 0.012Å, is in the range of high energy of γ-ray or X-ray.

Pair Annihilation

Pair Annihilation means the reverse process of pair production. In the pair annihilation, the electron and positron in the stationary state combine with each other and annihilate. Surely, the particles are disappeared and radiation energy will occur instead of two particles. For the momentum conservation, the most frequent process in pair annihilation is making two photons that have exactly opposite direction and the same amount of momentum. (Sometimes it produces three photons in the pair annihilation process.)

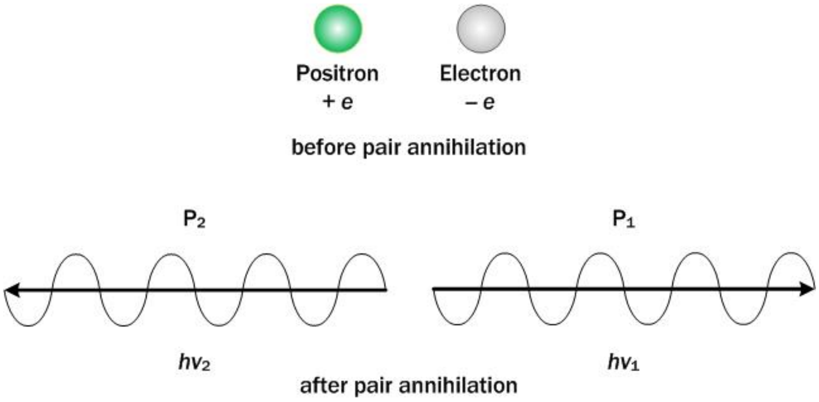


Figure 2 is shown the annihilation of pair electron and positron which is making two photons. In the case of Figure 2, the energy balance can be represented as:

$$K_- + K_+ + 2m_0c^2 = 2\,hv \text{ ---------eq.2}$$

K_- and K_+ represent the kinetic energy of the electron and positron before the collision. Also, $2m_0c^2$ means the rest mass energy of both particles. From the equation 2, if the initial kinetic energy was zero, then,

$$hv = m_0c^2 = 0.511 \text{ MeV ----------eq.3}$$

Therefore, in the equation 3, photons produced by pair annihilation have 0.511 MeV energy, and it correspond to 0.024Å of the wavelength in γ-ray. However, if the initial kinetic energy is not the same with zero, photon's energy is larger than 0.511 MeV, and its wavelength might be shorter than 0.024Å.

The positron is produced by the process of pair production. Generally, this positively-charged particle loses their energy by colliding with other particles in the path within matter, and finally combines with electron. We call those, "combined things", "positroium". The positronium collapse within about 10-10 second and produce two photons(pair annihilation).